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On robustness of Lyapunov-based nonlinear adaptive controllers

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Abstract: In this paper, we discuss robustness of a class of control Lyapunov function (CLF)-based nonlinear adaptive controllers with respect to input uncertainties. We prove that the adaptive controllers are robust with respect to monotone input nonlinearities. Moreover, we extend this result to the robust set-point regulation problem of nonlinear systems. The robustness of the controllers also confirmed by computer simulations.

Keywords: nonlinear control, adaptive control, sector nonlinearity, monotone nonlinearity

1. INTRODUCTION

Set-point regulation of nonlinear systems, e.g. robot manipulator control, is a fundamental and important problem for control applications. Unlike standard regulation problems, an operating point is not a equilibrium point of open-loop system; this implies system parameter uncertainties directly cause an offset error.

To achieve offset-free set-point regulation, adaptive control is one of the effective approach. This fact is well known in the field of robot control (see, e.g. Craig (2005), Siciliano et al. (2010)), and many adaptive controllers for robot manipulators are proposed; Craig et. al. (1987), Slotine and Li (1987), Sadegh and Horowitz (1990), Tomei (1991), Berghuis et. al. (1993), and so on. Note that all of these controllers have the same structure; the combinations of an adaptive parameter compensation term and a stabilization term. Thanks to the adaptive compensation term, the effect of gravity is precisely canceled even if parameter uncertainties exist.

For general input affine nonlinear systems, the above construction of adaptive controllers could be extended by employing control Lyapunov functions (CLFs). In the set-point regulation problem, a CLF is also available as an adaptive control Lyapunov function (ACLF) (see Krstić et al. (1995), Satoh et al. (2009)). This implies both adaptive compensating and stabilizing terms can be designed based on the CLF.

As well-studied in the robot manipulator control, the adaptive controllers are robust with respect to parameter uncertainties. On the other hand, robustness with respect to input uncertainties such as gain margins or sector margins (Grad (1987), Sepulchre et al. (1997)) also important in practice. In Satoh et al. (2009), the authors showed that the CLF-based adaptive controllers have gain margins if the stabilization term itself have gain margins. However, robustness results with respect to more general input uncertainties are not studied.

In this paper, we discuss robustness of the CLF-based adaptive controllers with respect to a class of nonlinear input uncertainties. In particular, we consider the monotone nonlinearity (Arcak and Kokotović (2001), Fan and Arcak (2003)) as the input uncertainties and discuss the stability of the perturbed closed loop systems.

2. PRELIMINARIES

In this section, we introduce basic definitions of mathematical terms and their fundamental properties.

Let us consider the following nonlinear system:

$$\dot{x} = f(x) + g(x)u \tag{1}$$

where $x \in \mathbb{R}^n$ is the state and $u \in \mathbb{R}^m$ the control input. We assume $f : \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{R}^n \to \mathbb{R}^{n\times m}$ are continuous mappings and $f(0) = 0$.

Control Lyapunov function (CLF) for (1) is defined as follows:

Definition 1. (control Lyapunov function). A C^1 function $V : \mathbb{R}^n \to \mathbb{R}$ is said to be a control Lyapunov function for (1) if the following properties holds:

(A1) V is proper; that is, the set $\{x \in \mathbb{R}^n | V(x) \le L\}$ is compact for every $L > 0$;
(A2) V is positive definite; that is, $V(0) = 0$ and $V(x) > 0$ for all $x \in \mathbb{R}^n \backslash \{0\}$;
(A3) the following holds:

$$\inf_{u\in\mathbb{R}^m} (L_f V + L_g V \cdot u) < 0,\ \forall x \in \mathbb{R}^n \backslash \{0\}, \tag{2}$$

where $L_f V$ and $L_g V$ are denote $(\partial V/\partial x)f(x)$ and $(\partial V/\partial x)g(x)$, respectively.

In this paper, we discuss the robustness of state feedback controllers with respect to input uncertainties. In nonlinear control theory, the following sector margins and gain

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margins are used to evaluate the robustness (Grad (1987), Sepulchre et al. (1997)):

Definition 2. (sector nonlinearity). A continuous mapping $\phi : \mathbb{R} \to \mathbb{R}$ is said to be a sector nonlinearity in $[\alpha, \beta]$, where $\alpha, \beta \in \mathbb{R}$ such that $0 < \alpha < 1 < \beta$, if the following conditions hold:

$$\begin{aligned} &\phi(0) = 0, \\ &\alpha u^2 \le u\phi(u) \le \beta u^2, \ \forall u \in \mathbb{R}\backslash\{0\}. \end{aligned} \tag{3}$$

Moreover, a mapping $\phi : \mathbb{R}^m \to \mathbb{R}^m; (u_1, \dots, u_m)^T \mapsto (\phi_1(u_1), \dots, \phi_m(u_m))^T$ is said to be a sector nonlinearity in $[\alpha, \beta]^m$ if each ϕ_i $(i = 1, \dots, m)$ is a sector nonlinearity in $[\alpha, \beta]$.

Definition 3. (sector margin). Let $k : \mathbb{R}^n \to \mathbb{R}^m$ be a given state feedback controller and $\phi : \mathbb{R}^m \to \mathbb{R}^m$ any sector nonlinearity in $[\alpha, \beta]^m$. Then, the controller $u = k(x)$ is said to have a sector margin $[\alpha, \beta]^m$ if the origin of the following closed loop system is asymptotically stable:

$$\dot{x} = f(x) + g(x)\phi(k(x)). \tag{4}$$

Gain margins are also defined as a special case of sector margins:

Definition 4. (gain margin). The controller $u = k(x)$ is said to have a gain margin $[\alpha, \beta]^m$ if the condition of Definition 3 holds for any ϕ satisfying

$$\begin{aligned} &\phi(u) = Ku, \ \forall u \in \mathbb{R}^m, \\ &K = \mathrm{diag}(\kappa_1, \dots, \kappa_m), \ \kappa_i \in [\alpha, \beta], \forall i \in \{1, \dots, m\}. \end{aligned} \tag{5}$$

Moreover, such ϕ is called a gain uncertainty in $[\alpha, \beta]^m$.

Remark 1. Sector and gain margins contain the asymptotic stability of the original system (1). This follows from the fact that $\phi(u) = u$ is both sector nonlinearity and a gain uncertainty in $[\alpha, \beta]^m$.

3. GAIN MARGINS OF CLF-BASED ADAPTIVE CONTROLLERS

In this paper, we consider the following perturbed system of (1):

$$\dot{x} = f(x) + g(x)(u - \theta), \tag{6}$$

where $\theta \in \mathbb{R}^m$ is a constant parameter.

The problem considered here is the asymptotic stabilization of $x = 0$ of (6). As mentioned in section 5, this problem is closely related to non-zero set-point regulation of system (1).

To consider CLF-based controller design, we introduce the following hypothesis:

Hypothesis 1. There exists a CLF $V(x)$ for nominal system (1) (i.e. system (6) with $\theta = 0$).

Then it is natural to construct a stabilizing state feedback for system (6) by

$$u = k(x) + \theta, \tag{7}$$

where $k : \mathbb{R}^n \to \mathbb{R}^m$ asymptotically stabilizes the origin of (1) and guarantees the sector margin $[\alpha, \beta]^m$ for some α and β such that $0 < \alpha < 1 < \beta$. Note that Such $k(x)$ always exists under the Hypothesis 1. For example, we can employ Sontag's universal formula (Sontag (1989)) as $k(x)$.

The controller (7) clearly asymptotically stabilizes the origin of (6). However, by this construction, the sector margin of $k(x)$ is lost. More precisely, controller (7) does not guarantees any sector/gain margin for system (6) despite $k(x)$ guarantees the sector margin for (1).

To "recover" the robustness of $k(x)$, we extend the controller (7) to the following adaptive control form:

$$u = k(x) + \hat{\theta}, \tag{8}$$

$$\dot{\hat{\theta}} = -\Gamma L_g V^T, \tag{9}$$

where $\hat{\theta}$ is a estimate of θ, $\dot{\hat{\theta}}$ its update law, and $\Gamma := \mathrm{diag}(\gamma_1, \dots, \gamma_m)$, $\gamma_i > 0$, $i \in \{1, \dots, m\}$ a adaptive gain matrix.

Remark 2. V is available for adaptive control design since V is also a adaptive control Lyapunov function (ACLF) for (6). For details on ACLF, refer to Krstić et al. (1995).

Remark 3. The controller (8)–(9) is available whether θ is known or not.

Importantly, the controller (8)–(9) guarantees a gain margin for (6). The following theorem is the generalization of Lemma 2 in Satoh et al. (2009).

Theorem 2. Let $\phi(u) = Ku$ be any gain uncertainty in $[\alpha, \beta]^m$. Then the closed loop system

$$\begin{aligned} \dot{x} &= f(x) + g(x)\left[K\left(k(x) + \hat{\theta}\right) - \theta\right] \\ \dot{\hat{\theta}} &= -\Gamma L_g V^T \end{aligned} \tag{10}$$

is asymptotically stable at $(x, \hat{\theta}) = (0, K^{-1}\theta)$.

Proof. Let $\theta_2 := K^{-1}\theta$. Consider the following Lyapunov function $\tilde{V}$ for (10):

$$\tilde{V}(x, \hat{\theta} - \theta_2) := V(x) + \frac{1}{2}(\hat{\theta} - \theta_2)^T K\Gamma^{-1}(\hat{\theta} - \theta_2). \tag{11}$$

Then the time derivative of $\tilde{V}$ is obtained as

$$\begin{aligned} \dot{\tilde{V}} &= \frac{\partial V}{\partial x}\left[f(x) + g(x)\left(K(k(x) + \hat{\theta}) - \theta\right)\right] \\ &\quad + (\hat{\theta} - \theta_2)^T K\Gamma^{-1}\dot{\hat{\theta}} \\ &= \frac{\partial V}{\partial x}\left(f(x) + g(x)Kk(x)\right) \\ &\quad + L_g V K\left(\hat{\theta} - \theta_2\right) - (\hat{\theta} - \theta_2)^T K\Gamma^{-1}\left(\Gamma L_g V^T\right) \\ &= \frac{\partial V}{\partial x}\left(f(x) + g(x)Kk(x)\right) \le 0. \end{aligned}$$

Note that the last inequality holds since $k(x)$ guarantees the gain margin. The convergence of x and $\hat{\theta}$ follows from the LaSalle's invariance principle (for more details, see the proof of Theorem 5 in section 4).

This theorem provides that the controller (8)–(9) is robust to gain uncertainties in $[\alpha, \beta]^m$. Is it possible to extend this result to more general input uncertainties ? We tackle this problem in the following section.

4. ROBUSTNESS WITH RESPECT TO MONOTONE UNCERTAINTIES

4.1 Monotone Nonlinearities

In Theorem 2, the key of the proof is that any gain uncertainty ϕ satisfies

$$\phi(k(x) + \hat{\theta}) = \phi(k(x)) + \phi(\hat{\theta}). \tag{12}$$

This implies the control input could be separated into the stabilization and parameter compensation terms. However, this property may not hold for general sector nonlinearities. Then we introduce the following monotone nonlinearity as a "separable" class of sector nonlinearities.

Definition 5. (monotone nonlinearity). A sector nonlinearity $\phi : \mathbb{R} \to \mathbb{R}$ in $[\alpha, \beta]$ is said to be a monotone nonlinearity in $[\alpha, \beta]$ if the following condition holds:

$$\phi(u + v) = \psi(v, u) + \phi(u), \forall u, v \in \mathbb{R}, \tag{13}$$

where $\psi : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is a continuous mapping such that $\psi(\cdot, u)$ is a sector nonlinearity in $[\alpha, \beta]$ for each fixed $u \in \mathbb{R}\backslash\{0\} and \psi(\cdot, 0) = 0$.

A monotone nonlinearity $\phi : \mathbb{R}^m \to \mathbb{R}^m$ in $[\alpha, \beta]^m$ is also defined in the same manner as the sector nonlinearity in $[\alpha, \beta]^m$ (see Definition 2).

Monotone nonlinearities are, for example, introduced in nonlinear observer design problem (see Arcak and Kokotović (2001), Fan and Arcak (2003)). The following lemma shows that Definiton 5 is a direct extension of the Definition in Arcak and Kokotović (2001):

Lemma 3. Let $\phi : \mathbb{R} \to \mathbb{R}$ be a C^1 sector nonlinearity in $[\alpha, \beta]$. Then the condition (13) is equivalent to

$$\alpha \le \frac{d\phi}{du}(u) \le \beta, \ \forall u \in \mathbb{R}. \tag{14}$$

Proof.

(i) (13) $\Rightarrow$ (14) :

Since ψ in (13) is a sector nonlinearity in $[\alpha, \beta]$, the following holds:

$$\alpha \le \frac{\phi(u + v) - \phi(u)}{v} \le \beta, \ \forall u \in \mathbb{R}, \ v \in \mathbb{R}\backslash\{0\}. \tag{15}$$

Moreover, $\lim_{v\to 0}(\phi(u + v) - \phi(u))/v$ exists since ϕ is C^1. Then (14) holds.

(ii) (14) $\Rightarrow$ (13) :

It is sufficient to show

$$\alpha v^2 \le v(\phi(u + v) - \phi(u)) \le \beta v^2 \ \forall v \in \mathbb{R}\backslash\{0\}. \tag{16}$$

According to the mean value theorem, there exists $w \in \mathbb{R}$ such that

$$v\left(\phi(u + v) - \phi(u)\right) = v^2 \frac{\partial\phi}{\partial u}(w). \tag{17}$$

By (14), $\alpha \le (\partial\phi/\partial u)(w) \le \beta$ holds and we can obtain (16).

Note that any monotone nonlinearity ϕ and its inverse ϕ^{-1} are bijective:

Lemma 4. Any monotone nonlinearity $\phi : \mathbb{R} \to \mathbb{R}$ in $[\alpha, \beta]$ is bijective.

Proof. The surjectivity follows from

$$\lim_{u\to\pm\infty} \phi(u) = \pm\infty. \tag{18}$$

Let $u, v \in \mathbb{R}$ be such that $\phi(u) = \phi(v)$ and $u \ne v$, and let $w := u - v$. Then,

$$\phi(u) = \phi(v + w) = \psi(w, v) + \phi(v) \tag{19}$$

holds and $\psi(w, v) = 0$ is derived. Since $w \ne 0$, this contradicts the assumption that $\psi(\cdot, v)$ is a sector nonlinearity.

Example 1. Let us consider the following C^1 mapping $\phi : \mathbb{R} \to \mathbb{R}$:

$$\phi(u) = \begin{cases} e^u + \dfrac{u}{2} - 1 & (u \ge 0) \\ -e^{-u} + \dfrac{u}{2} + 1 & (u < 0) \end{cases}. \tag{20}$$

ϕ is a sector nonlinearity in $[1/2, \infty]$ since $\phi(0) = 0$ and $u\phi(u) \ge (1/2)u^2, \ \forall u \ne 0$ hold. Moreover, ϕ is a monotone nonlinearity in $[1/2, \infty]$ since the following holds:

$$\frac{\partial\phi}{\partial u}(u) = \begin{cases} e^u + \dfrac{1}{2} & (u \ge 0) \\ e^{-u} + \dfrac{1}{2} & (u < 0) \end{cases}. \tag{21}$$

4.2 Main Theorem

The following theorem is the main result of this paper.

Theorem 5. Let $\phi : \mathbb{R}^m \to \mathbb{R}^m$ be any monotone nonlinearity in $[\alpha, \beta]^m$. Then, the closed loop system

$$\begin{aligned} \dot{x} &= f(x) + g(x)\left[\phi\left(k(x) + \hat{\theta}\right) - \theta\right] \\ \dot{\hat{\theta}} &= -\Gamma L_g V^T, \end{aligned} \tag{22}$$

is asymptotically stable at $(x, \hat{\theta}) = (0, \phi^{-1}(\theta))$.

Proof. Let $\theta_2 := \phi^{-1}(\theta) = [\theta_{2_1}, \ldots, \theta_{2_m}]^T$ and $\hat{\theta} = [\hat{\theta}_1, \ldots, \hat{\theta}_m]^T$. Since ϕ is a monotone nonlinearity, there exists $\psi = [\psi_1, \ldots, \psi_m]^T$ such that each ψ_i satisfies (13). Note that the second term of the first equation of (22) is separable into

$$\begin{aligned} &g(x)\left(\phi(k(x) + \hat{\theta}) - \theta\right) \\ &= g(x)\left(\psi(k(x), \hat{\theta}) + \phi(\hat{\theta}) - \phi(\theta_2)\right) \\ &= g(x)\left(\psi(k(x), \hat{\theta}) + \psi(\hat{\theta}_2 - \theta_2, \theta_2)\right). \end{aligned} \tag{23}$$

To cancel the term $g(x)\psi(\hat{\theta}_2 - \theta_2, \theta_2)$, we employ the following Lurie-type Lyapunov function:

$$\tilde{V}(x, \hat{\theta} - \theta_2) = V(x) + \sum_{i=1}^{m} \frac{1}{\gamma_i} \int_0^{\hat{\theta}_i - \theta_{2_i}} \psi_i(\sigma, \theta_{2_i}) d\sigma. \tag{24}$$

Note that $\tilde{V}$ is positive definite and proper since each $\psi_i(\cdot, \theta_{2_i})$ is a sector nonlinearity. By using (23), the derivative of $\tilde{V}$ is calculated as

$$\begin{aligned} \dot{\tilde{V}} &= \frac{\partial V}{\partial x}\left[f(x) + g(x)\left(\psi(k(x), \hat{\theta}) + \psi(\hat{\theta}_2 - \theta_2, \theta_2)\right)\right] \\ &\quad + \psi^T(\hat{\theta} - \theta_2, \theta_2)\Gamma^{-1}\dot{\hat{\theta}} \\ &= \frac{\partial V}{\partial x}\left(f(x) + g(x)\psi(k(x), \hat{\theta})\right) \\ &\quad + \frac{\partial V}{\partial x}\left(g(x)\psi(\hat{\theta}_2 - \theta_2, \theta_2)\right) - \psi^T(\hat{\theta} - \theta_2, \theta_2)L_g V^T \\ &= \frac{\partial V}{\partial x}\left(f(x) + g(x)\psi(k(x), \hat{\theta})\right) \le 0. \end{aligned}$$

The last inequality follows from the fact that $k(x)$ guarantees the sector margin $[\alpha, \beta]^m$.

Then we prove the state and parameter convergence. Let

$$\begin{aligned} S :&= \left\{(x, \hat{\theta} - \theta_2) \mid \tilde{V}(x, \hat{\theta} - \theta_2) = 0\right\} \\ &= \{(0, \hat{\theta} - \theta_2) \mid \hat{\theta} \in \mathbb{R}^m\}. \end{aligned} \tag{25}$$

Note that the largest invariant set contained in S is $\{(0,0)\}$ since the following holds:

$$\begin{aligned} x \equiv 0 \ \Rightarrow \ & f(0) + g(0)\left(\phi(k(0)+\hat{\theta}) - \theta\right) \equiv 0 \\ \Rightarrow \ & \phi(\hat{\theta}) \equiv \theta \ \Rightarrow \ \hat{\theta} \equiv \theta_2. \end{aligned} \tag{26}$$

Hence $(x, \hat{\theta} - \theta_2) = (0,0)$ is asymptotically stable by LaSalle's invariance principle (Krstić et al. (1995)).

5. ROBUST SET-POINT REGULATION OF NONLINEAR SYSTEMS

In this section, we consider the robust set-point regulation problem of system (1), as an application of Theorem 5.

Let $x_o \in \mathbb{R}^n$ be a given non-zero operating point. We assume that there exists a input $u_o \in \mathbb{R}^m$ such that

$$f(x_o) + g(x_o)u_o = 0. \tag{27}$$

We define the error variable $e := x - x_o$, and consider the error dynamics given by

$$\dot{e} = f(e + x_o) + g(e + x_o)u. \tag{28}$$

By separating the input u as $u = u + u_o - u_o$, we can obtain the following system:

$$\dot{e} = \tilde{f}(e) + \tilde{g}(e)(u - u_o), \tag{29}$$

where

$$\tilde{f}(e) := f(e + x_o) + g(e + x_o)u_o, \tag{30}$$

$$\tilde{g}(e) := g(e + x_o). \tag{31}$$

Note that $\tilde{f}(0) = 0$ is achieved by the above input separation. Now, the problem is reduced to the asymptotic stabilization of $e = 0$ of (29). Moreover, (29) has the same structure as the system (6) considered in the previous section; u_o correspond to the constant parameter θ.

Hypothesis 6. There exists a CLF $V(e)$ for

$$\dot{e} = \tilde{f}(e) + \tilde{g}(e)u. \tag{32}$$

Then, the following robust set-point regulation result is obtained as a corollary of Theorem 5:

Corollary 7. Consider the following adaptive controller for (29):

$$u = k(e) + \hat{u}_o, \tag{33}$$

$$\dot{\hat{u}}_o = -\Gamma L_{\tilde{g}} V(e), \tag{34}$$

where $k : \mathbb{R}^n \to \mathbb{R}^m$ is an asymptotic stabilizing state feedback for (32) which guarantees the sector margin $[\alpha, \beta]^m$, $\hat{u}_o$ an estimate of u_o, and $\dot{\hat{u}}_o$ its update law. Moreover, $\phi : \mathbb{R}^m \to \mathbb{R}^m$ be any monotone nonlinearity in $[\alpha, \beta]^m$. Then, the perturbed closed loop system

$$\begin{aligned} \dot{e} &= \tilde{f}(e) + \tilde{g}\left(\phi(k(e) + \hat{u}_o) - u_o\right), \\ \dot{\hat{u}}_o &= -\Gamma L_{\tilde{g}} V(e), \end{aligned} \tag{35}$$

is asymptotically stable at $(e, \hat{u}_o) = (0, \phi^{-1}(u_o))$.

Example 2. Let us consider the following nonlinear system:

$$\begin{cases} \dot{x}_1 &= -x_1 + 2x_2 \\ \dot{x}_2 &= -x_1^2 + u. \end{cases} \tag{36}$$

Here we consider the set-point regulation of (36) for operating point of the form $x_o := [a, a/2]^T$, $a \in \mathbb{R}\backslash\{0\}$. By (27), the input u_o is obtained as

$$u_o = a^2. \tag{37}$$

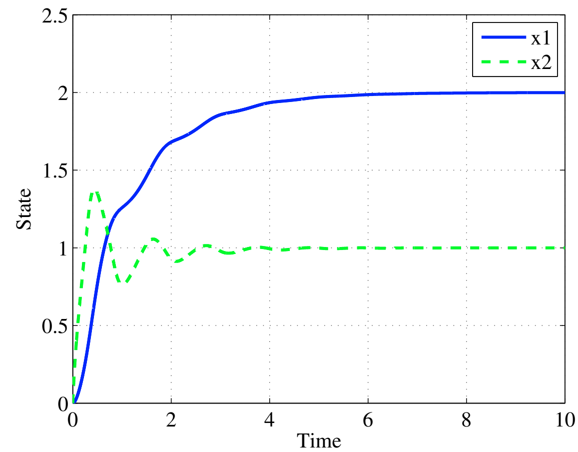

Fig. 1. Nominal system: time response of the state.

Then the dynamics of $e := [x_1 - a, x_2 - a/2]^T$ is

$$\begin{cases} \dot{e}_1 &= -e_1 + 2e_2 \\ \dot{e}_2 &= -(e_1 + a)^2 + a^2 + (u - u_o) \end{cases}. \tag{38}$$

We can employ the following CLF for controller design:

$$V(e) = \frac{1}{2}(e_1^2 + e_2^2). \tag{39}$$

$L_{\tilde{f}}V(e)$ and $L_{\tilde{g}}V(e)$ are calculated as

$$L_{\tilde{f}}V(e) = e_1(-e_1 + 2e_2) - e_2(e_1 + a)^2 + a^2 e_2, \tag{40}$$

$$L_{\tilde{g}}V(e) = e_2. \tag{41}$$

Then we can construct the adaptive controller (33)–(34). We employ the following Sontag-type controller as $k(e)$:

$$k(e) = \begin{cases} -\dfrac{L_{\tilde{f}}V + \sqrt{(L_{\tilde{f}}V)^2 + c(L_{\tilde{g}}V)^4}}{L_{\tilde{g}}V} & (L_{\tilde{g}}V \neq 0) \\ 0 & (L_{\tilde{g}}V = 0) \end{cases}, \tag{42}$$

where $c > 0$ is a control parameter. Note that this parameter does not cause any problem for stability and the sector margin of original Sontag's universal formula.

The initial state and the operating point are $x_o = [2, 1]^T$ ($a = 2$) and $e(0) = [-2, -1]^T$ ($x(0) = [0, 0]^T$). We set the control parameters as $c = 50$, $\gamma = 30$, and $\hat{u}_o(0) = 0$.

Firstly, we apply the controller to nominal system (45). The results of simulation are shown in Figs. 1–3. In Fig. 1, both x_1 and x_2 converges to the desired operating point. Moreover, we can see the control input and the estimate of $\hat{u}_o$ converges to the real value $u_o = a^2 = 4$.

Then we introduce the input uncertainty for (45). Let us consider the following monotone nonlinearity ϕ belongs to $[1/2, \infty]$:

$$\phi(u) = \frac{u}{2} + \frac{u^2}{2(u^2+1)}\mathrm{sgn}\,(u). \tag{43}$$

Simulation results with ϕ are depicted in Figs. 4–6. In Fig. 4, we can see the both states successfully converges to the operating point. The state convergence speed is slightly slow in comparison with Fig. 1. The control input converges to 4.0 in Fig. 5 as similar to the nominal case.

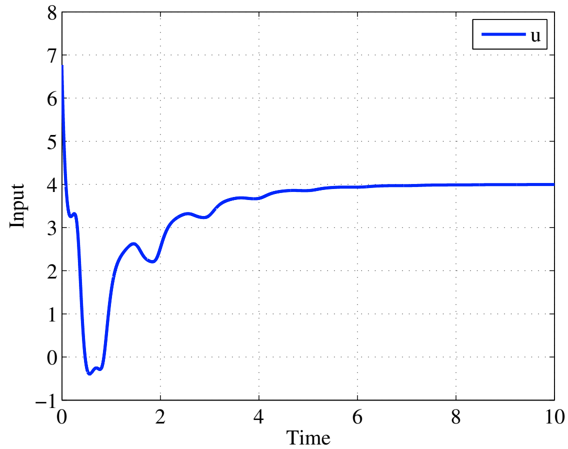

Fig. 2. Nominal system: time response of the input.

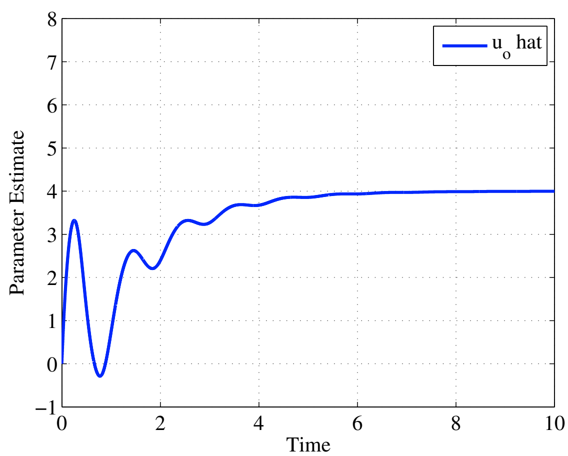

Fig. 3. Nominal system: Time response of the parameter estimate.

However, in Fig. 6, $\hat{u}_o$ seems to converge 7.0. This is due to the effect of the input uncertainty ϕ. Actually, $\hat{u}_o(10.0) \simeq 7.0047$ and $\phi(\hat{u}_o(10.0)) \simeq 3.9924$ holds.

This result indicates that the adaptive parameter compensation is robust with respect to monotone input uncertainties.

6. REMARKS ON SECTOR MARGINS

As discussed in the section 4.1, the key of our robustness results is the separable property (13). However, sector nonlinearities may not satisfy this property and then it seems to be difficult to guarantee sector margins.

Due to Lemma 3, this property is interpreted as a kind of strictly increasing properties; and sector nonlinearities may have "negative slope" around u_o defined by (27). Then we are interested in the following question: is this "negative slope" causes an offset error ?

To study this problem, we consider the set-point regulation of the following one-dimensional linear system:

$$\dot{x} = x + u. \tag{44}$$

We set $x_o = -\pi/4$ and then $e = x + \pi/4$. The error dynamics is given by

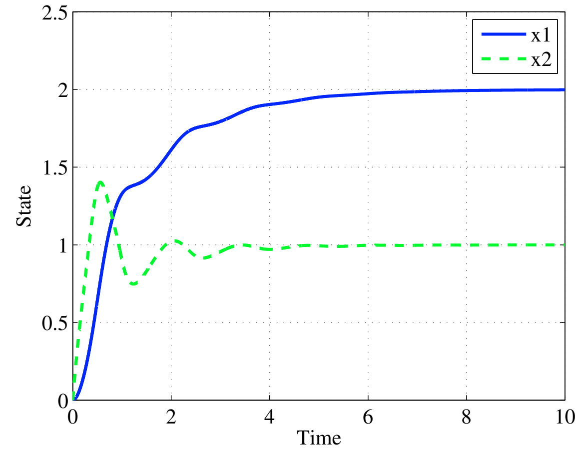

Fig. 4. perturbed system: Time response of the state.

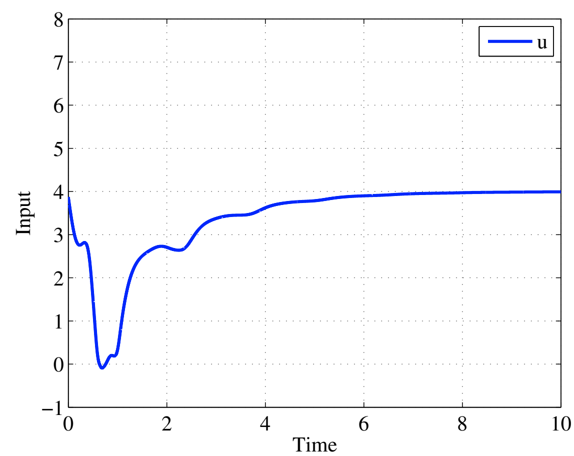

Fig. 5. Perturbed system: time response of the input.

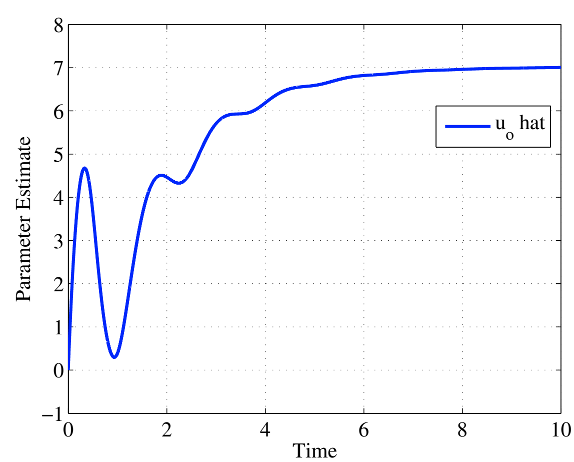

Fig. 6. Perturbed system: time response of the parameter estimate.

$$\dot{e} = e + \left(u - \frac{\pi}{4}\right). \tag{45}$$

Then the perturbed system is

$$\dot{e} = e + \left(\phi(u) - \frac{\pi}{4}\right), \tag{46}$$

where we employ the following sector nonlinearity ϕ:

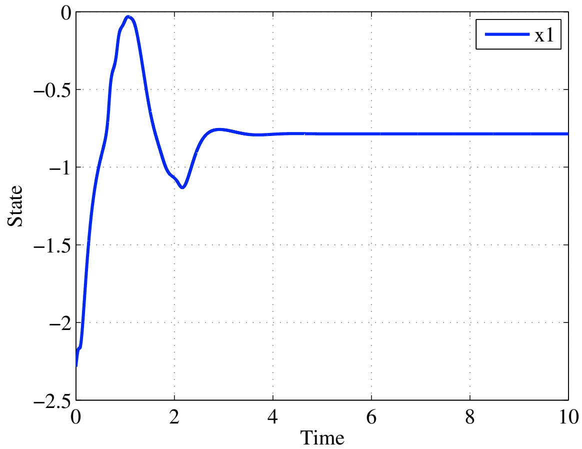

Fig. 7. Perturbed system (46): time response of the state.

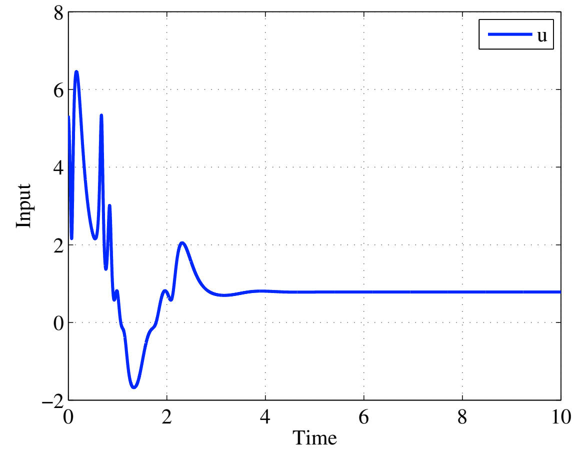

Fig. 8. Perturbed system (46): time response of the input.

$$\phi(u) := \left(1 + \frac{1}{2}\sin(4u)\right) u. \tag{47}$$

The linear approximation of system (46) around $(e, u) = (0, \pi/4)$ is obtained as

$$\dot{e} = e + \left[\left(1 - \frac{\pi}{2}\right) u - \frac{\pi}{4}\right]. \tag{48}$$

Since $(1 - \pi/2) < 0$, ϕ has a negative slope around $u = 0$. This implies $x = -\pi/4$ is locally unstable when a stabilization term $k(x)$ designed for (45) is used.

We apply the proposed controller (33)–(34) to the perturbed system (46). A CLF for (45) is

$$V(e) = \frac{1}{2}e^2. \tag{49}$$

Then we can design the following adptive controller for (45):

$$u = k(e) + \hat{u}_o = -(1 + \sqrt{2})e + \hat{u}_o, \tag{50}$$

$$\dot{\hat{u}}_o = -\gamma e, \tag{51}$$

where $k(e)$ is a Sontag type controller (42) with $c = 1$. We set $x(0) = -\pi/4 - 1.5$ ($e(0) = -1.5$), $\gamma = 10$, and $\hat{u}_o(0) = 0$. Simulation results are shown in Figs. 7 and 8. We can see the state successfully converges even if the negative slope exists. This simulation results show the potential of the adaptive parameter compensation for guaranteeing sector margins.

7. CONCLUSION

In this paper, we discussed the robustness of a class of CLF-based adaptive controllers with respect to input uncertainties. We showed that the adaptive controllers are robust with respect to any monotone input nonlinearity. This result was applied to the robust set-point regulation problem of nonlinear systems. The robustness of the controllers is also confirmed by computer simulation. The further analysis of sector margins remains for our future work.

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